ON THE POLARIZATION OF RADIO-WAVE TRAVELLING THROUGH THE IONOSPHERE

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ABSTRACT. The limiting polarization as deduced by Baker and Green without taking the usual Appleton-Hartree formulae has been shown to be identical with the limiting polarization deduced directly from the Appleton-Hartree formula. From Bailey's formulae for the amplitude-ratio of the normal to the abnormal components of the magnetic vector of the radio-wave, the phase-difference between them at any level of the ionosphere has been deduced for $\nu << \nu_c$ where ν_c is the critical collisional frequency. It has also been shown that for $\nu << \nu_c$, the amplitude ratio is nearly unaffected at any level by electron collisional frequency

1. *Identity of the expression for the limiting polarization as given by Baker and Green with that deduced directly from the Appleton-Hartree formulae*

Without taking the usual Appleton-Hartree formulae (1927, 1929) it was shown by Baker and Green (1933) that the limiting polarization R_a is given by the roots of the following equation.

$$R_a^2+\frac{l^2}{n}q'R_a-1=0 \tag{1}$$

where

$$R_a=-\frac{E_y}{jE_t},\quad \frac{l^2}{n}=\frac{\sin^2\theta}{\cos\theta}\quad q'=\frac{p_0}{p-j\nu},\quad p_0=\frac{eH}{mc} \tag{2}$$

$\theta =$ angle between the direction of propagation of the radio-wave and the positive direction of earth's magnetic field.

$E_y =$ component of the electric vector along OY (Fig. 1).

$E_t =$ component of the electric vector along OT (Fig. 1).

$H =$ intensity of earth's magnetic field.

$e, m =$ electronic charge and mass.

$\nu =$ electronic collisional frequency.

$c =$ velocity of light in vacuum.

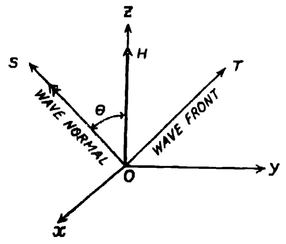

Fig. 1. Co-ordinate system used by Baker and Green. OS—wavenormal
OT—wavefront
H—earth's magnetic field

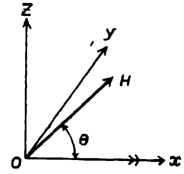

Fig. 2. Right-handed co-ordinate system used by Appleton
OX—wave normal
H—earth's magnetic field

When the right-handed co-ordinate system of Fig. 2 is used, where the direction of propagation of the radio-wave has been taken along OX-axis, it can be shown that

$$R_a = \frac{E_y}{jE} = -j\left(\frac{hz}{hy}\right) \tag{3}$$

where

hz = component of the magnetic vector along OZ (Fig. 2).
hy = component of the magnetic vector along OY (Fig. 2).

From Eq. (3)

$$\left(\frac{hz}{hy}\right) = jR_a \tag{3a}$$

Solving Eq. (1), we have

$$R_a = -\frac{l^2}{2n}q' \pm \sqrt{\frac{l^4}{4n^2}q'^2 + 1} \tag{1a}$$

Putting the value of l^2/n and q' from Eq. (2) in Eq. (1a)

$$R_a = -\frac{eH\sin^2\theta}{2mc\cos\theta(p-j\nu)} \pm \sqrt{\frac{e^2H^2\sin^4\theta}{4m^2c^2\cos^2\theta(p-j\nu)^2} + 1} \quad \ldots \tag{1b}$$

Since the critical collisional frequency is

$$\nu_c = \frac{p_H}{2}\frac{\sin^2\theta}{\cos\theta}, \text{ where } p_H = \frac{eH}{mc}$$

we get

$$R_a = -\frac{\nu_c}{p-j\nu} \pm \sqrt{\frac{\nu_c^2}{(p-j\nu)^2} + 1} \tag{1c}$$

Using (1c) and (3a) :

$$\begin{pmatrix} hz \\ hy \end{pmatrix} = j\left[-\frac{\nu_e}{p-j\nu} \pm \sqrt{\frac{\nu_e^2}{(p-j\nu)^2}+1}\,\right] \qquad \ldots \quad (1d)$$

This equation can be deduced *directly* from the Appleton-Hartree formula. Appleton used the right-handed co-ordinate system of Fig 2 and the polarization is given by

$$\begin{pmatrix} hz \\ hy \end{pmatrix} = R = -\frac{j}{\gamma_L}\left[-\frac{\gamma_T^2}{2(1+\alpha+j\beta)} \pm \sqrt{\frac{\gamma_T^4}{4(1+\alpha+j\beta)^2}+\gamma_L^2}\,\right] \qquad \ldots \quad (4)$$

Using $\alpha = p^2/p_0^2$, $\beta = p\nu/p_0^2$, $p_0^2 = 4\pi Ne^2/m$, $\gamma_{L,T} = pp_{L,T}/p_0^2$
Eq (4) can be written as

$$R = -\frac{j}{p_L}\left[-\frac{pp_T^2}{2(p_0^2-p^2+jp\nu)} \pm \sqrt{\frac{p^2p_T^4}{4(p_0^2-p^2+jp\nu)^2}+p_L^2}\,\right] \qquad \ldots \quad (4a)$$

Hence the limiting value is obtained by putting $p_0^2 = 0$ (i e. $N = 0$) in Eq. (4a)

$$R = \begin{pmatrix} hz \\ hy \end{pmatrix} = j\left[-\frac{\nu_e}{p-j\nu} \pm \sqrt{\frac{\nu_e^2}{(p-j\nu)^2}+1}\,\right] \qquad \ldots \quad (4b)$$

It is seen that Eqs (1d) and (4b) are identical.

2. *Evaluation of the phase-difference between the normal and the abnormal Components of the magnetic vector of a radio-wave at any level of the ionosphere for* $\nu << \nu_e$

It was shown by Bailey (1934) that the amplitude-ratios of the normal to the abnormal components of magnetic vector are given by

$$\rho_0 = a\left[1 - \frac{d_1}{d_2}\cot\phi_0\right] \text{ for the } O\text{-mode} . \qquad \ldots \quad (5a)$$

and

$$\rho_x = a\left[1 + \frac{d_1}{d_2}\cot\phi_0\right] \text{ for the } X\text{-mode} . \qquad \ldots \quad (5b)$$

where

$$a = \tfrac{1}{2}[\sqrt{1+Y} + \sqrt{1+Y'}]$$

$$Y = 2\frac{d_1 d_2^2}{d_1^2+d_2^2} + \frac{d_1^2 d_2^2}{d_1^2+d_2^2} \qquad \ldots \quad (5c)$$

$$Y' = -2\,\frac{d_1 d_2^2}{d_1^2+d_2^2} + \frac{d_1^2 d_2^2}{d_1^2+d_2^2}$$

$$d_1 = \nu_c/\nu,\ d_2 = \nu_c/p',\ p' = p\left(1-\frac{p_0^2}{p^2}\right),\ p_0^2 = \frac{4\pi Ne^2}{m}$$

ϕ_0 = phase-difference for the O-mode.

Using Eq. (5c), it can be shown ·

$$Y = \frac{\nu_c}{\nu^2+p'^2}(2\nu+\nu_e) \quad \dots \text{(6a)}$$

$$Y' = \frac{\nu_c}{\nu^2+p'^2}(\nu_c-2\nu) \quad \dots \text{(6b)}$$

Using Eqs. (5a) and (5b) and the relation, $\rho_0\rho_x = 1$,

$$\cot\phi_0 = \pm\frac{d_2}{d_1}\sqrt{1-\frac{1}{a^2}} \quad \dots \text{(7a)}$$

When $\nu << \nu_c$, we get from (6a), (6b) and (5c),

$$Y = Y' \simeq \frac{\nu_e^2}{\nu^2+p'^2} \quad \dots \text{(6c)}$$

Hence
$$a^2 \simeq \frac{p'^2+\nu_c^2}{p'^2+\nu^2} \quad \dots (6d)$$

Using (6d) and (7a)

$$\tan\phi_0 \simeq \pm\frac{p'}{\nu}\sqrt{1+\frac{p'^2}{\nu_c^2}} \quad \dots \text{(7b)}$$

According to Murty and Khastgir (1960), ϕ_0 lies in the first quadrant for the vertically downcoming wave in the northern hemisphere, hence taking the positive sign in (7b), we have

$$\tan\phi_0 \simeq \frac{p'}{\nu}\sqrt{1+\frac{p'^2}{\nu_c^2}} \quad \dots \text{(7c)}$$

Similarly, for the vertically downcoming radio wave in the southern hemisphere :

$$\tan\phi_0 \simeq -\frac{p'}{\nu}\sqrt{1+\frac{p'^2}{\nu_c^2}} \quad \dots (7d)$$

From Eq. (7c), we can draw several conclusions .

(a) At the level $p_0^2 = p^2$ (i.e. $p' = 0$), the wave is plane-polarised.

(b) The sense of rotation of the magnetic vector is reversed when the wave crosses the level $p_0^2 = p^2$.

(c) Since at the lower boundary of the ionosphere, $p' = p$ and at the level, $p_0^2 = p^2$, $p' = 0$, the phase-difference gradually increases from zero to a certain value depending on p, ν, and ν_c as the wave comes down from the level, $p_0^2 = p^2$, to the lower boundary of the ionosphere.

(d) At the level $p_0^2 = p^2 - pp_H$,

$$[\tan \phi_0]_{p_0^2 = p^2 - pp_H} = \frac{p_H}{\nu}[\cot^2\theta + \operatorname{cosec}^2\theta]$$

3. *Effect of electron collisional frequency on the amplitude-ratio at any level for* $\nu << \nu_c$.

It can be shown from the Appleton-Hartree formulae (1927, 1929) that the amplitude-ratio for the ordinary mode for vertically down-coming radio-wave in the northern hemisphere for zero collisional frequency is given by

$$\rho_0 = -\frac{\nu_c}{p'} + \sqrt{1 + \frac{\nu_c^2}{p'^2}} \tag{8}$$

where

$$p' = p\left(1 - \frac{p_0^2}{p^2}\right), \quad \nu_c = p_H \sin^2\theta/2 \cos\theta$$

From Eq. (8), it can be shown .

$$\left[\frac{1+\rho_0^2}{1-\rho_0^2}\right]_{\nu=0} = \frac{\sqrt{\nu_c^2 + p'^2}}{\nu_c} \tag{9}$$

It has been shown by Murty and Khastgir (1959) that

$$\rho_0 = \frac{a'}{\nu_c}\left[\frac{\nu}{\cos\phi_0} - \frac{p'}{\sin\phi_0}\right] \tag{10a}$$

and

$$\rho_x = \frac{a'}{\nu_c}\left[\frac{\nu}{\cos\phi_0} + \frac{p'}{\sin\phi_0}\right] \tag{10b}$$

where

$$a' = \frac{\nu_c}{\nu^2 + p'^2}$$

Using $\rho_0\rho_x = 1$, we have from Eqs. (10a), (10b)

$$\rho_0^2 = \frac{\dfrac{\nu}{\cos\phi_0} - \dfrac{p'}{\sin\phi_0}}{\dfrac{\nu}{\cos\phi_0} + \dfrac{p'}{\sin\phi_0}} \qquad \ldots \quad (10c)$$

Hence from Eq. (10c)

$$\left[\frac{1+\rho_0^2}{1-\rho_0^2}\right]_{\nu \quad 0} = \frac{\nu}{p'}\tan\phi_0 \qquad \ldots \quad (11)$$

Using (11), (7c),

$$\left[\frac{1+\rho_0^2}{1-\rho_0^2}\right] \qquad \sqrt{\nu_c^2 + p'^2} \qquad \ldots \quad (11a)$$

and from (11a) and (9),

$$[\rho_0]_{\nu \neq 0} \simeq [\rho_0]_{\nu=0}$$

This means that at any level of the ionosphere, the amplitude-ratio for the *O*-mode is nearly unaffected by electron collisional frequency provided the critical collisional frequency is much larger than electron-collisional frequency. The same conclusion can be drawn for **X**-mode also.

REFERENCES

Appleton, E. V., 1927, U.R.S.I. Paper.

Bailey, V. A., 1934, *Phil. Mag.*, **19**, 376

Baker, W. G. and Green, A. L., 1933, *Proc. I R E*, **21,** 1103.

Hartree, D. R., 1929, *Proc. Camb. Phil. Soc.*, **25**, 47

Murty, Y. S. N. and Khastgir, S. R., 1959, *Proc. Nat. Inst. sci*, **25A**, No. 5

Murty, Y. S. N. and Khastgir, S. R., 1960, *Jour. Geo. Res.*, **65**, May Issue